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LINEAR FILTERING FOR SHIPBOARD MAD RECORDERS.(U)
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LINEAR FILTERING FOR SHIPBORNE MAD RECORDERS

Summary

This report describes the derivation of linear filters for MAD equipment to be used for submarine detection from high-speed surface vessels. Since the bandwidth required to pass all the anomaly signals of interest is considerably larger than in airborne systems, it is recommended that the passband be covered by two filters, one driving each pen of a two-channel recorder. It is also recommended that each of these filters be provided with switchable cutoff frequencies for improved noise discrimination whenever the target's closing rate can be estimated. Furthermore, a selection of variable-frequency notch filters should be available to cascade with the basic filters in order to remove quasi-periodic noise, both that of geomagnetic origin as well as periodic maneuver noise.

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1. Introduction

The class of signals to be considered is that generated by passing through the field of a magnetic dipole with speeds-along-track, v, between 20 and 70 knots and smallest slant ranges, z, between 300 and 1500 feet (figure 1).

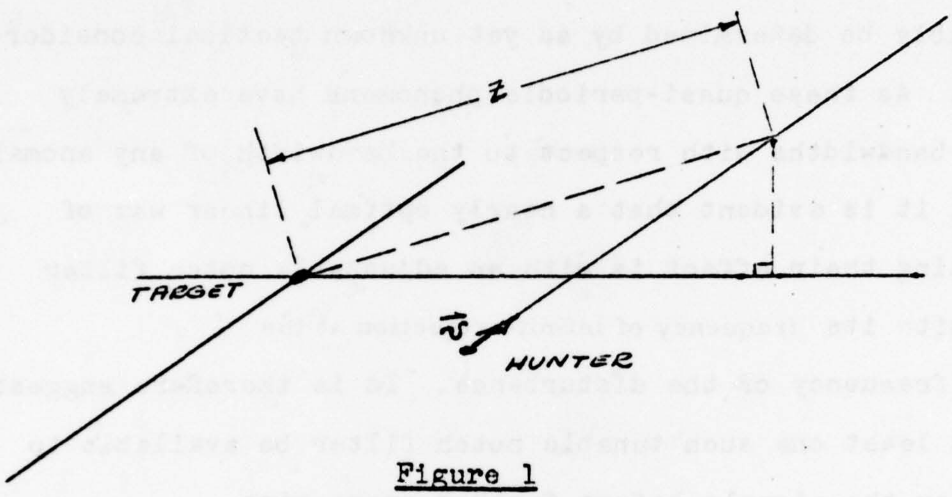


Figure 1

Additive noise corrupts the observed signal. This noise is assumed to be the sum of a stationary Gaussian component with power spectral density

$$N(\omega) = N_o/\omega^2 \tag{1.1}$$

within the passband of the signal collection, and a nonstationary component comprising a random sequence of quasi-periodic wave-trains caused both by natural geomagnetic phenomena ([1], pp. 7-55, [2], p. 287ff) and by periodic maneuvers (trapping circles, cloverleaves, etc. ([1], pp. 67-71)).

Filters are required to discriminate against the noise while preserving the shape (in time) of the anomaly signals

as closely as possible. Although the center frequency of naturally occurring quasi-periodic noise is relatively stable once the wavetrain has begun, this frequency is unpredictable with any precision in advance of the wavetrain's commencement; similarly, the center frequency of periodic maneuver noise will presumably be determined by as yet unknown tactical considerations. As these quasi-periodic phenomena have extremely narrow bandwidths with respect to the bandwidth of any anomaly signal, it is evident that a nearly optimal linear way of minimizing their effect is with an adjustable notch filter tuned with its frequency of infinite rejection at the center frequency of the disturbance. It is therefore suggested that at least one such tunable notch filter be available to prefilter the signals before further processing.

The remainder of this report is concerned with filtering against the stationary Gaussian component of the noise. Since the bandwidth of this noise is large compared to the bandwidth of an anomaly signal, it cannot simply be stripped away with a narrow-band filter. It is intuitively clear that the optimal linear filter will be one whose response at any frequency is directly related to the ratio of signal power to noise power at that frequency; this notion will be made more precise later on. It follows immediately, however, that since the noise power is not equally distributed among all frequencies (equation (1.1)) it is unlikely that the filter will have con-

stant gain over the passband of an anomaly signal. Therefore some waveform degradation is inevitable; indeed, it can be shown that the optimal linear filter for a single signal in noise with spectral density given by equation (1.1) is a "matched filter" for the signal's derivative, whose output (in the absence of noise) is the first derivative of the autocorrelation function of the signal. Conversely, a filter that preserves signal waveshape exactly can remove no noise at all.

In the sections of the report to follow, filters will be designed based on the theory of optimal linear filtering due to N. Wiener and A. N. Kolmogorov. It will unfortunately turn out that the filters so specified are unrealizable with the customary physical components, and must be approximated. In the interests of simplicity, ease of actual synthesis, and freedom from critical adjustment, it was decided to make the approximation with physically realizable filters having only real poles and zeros.

2. Development of a mathematical model

It has been shown ([1], p. 137) that if the vector closing velocity $\vec{v}$ (figure 1) is constant, than any anomaly signal can be represented as a linear combination of three linearly independent basis signals

$$s(t) = a_0 s_0(t) + a_1 s_1(t) + a_2 s_2(t) \tag{2.1}$$

in which the coefficients $\{a_j\}_0^2$ depend in a complicated fashion

on geographical location, dipole orientation and strength, and other time-independent physical parameters. The three basis signals are given by

$$\begin{aligned} s_0(t) &= (\theta^2+1)^{-5/2} \\ s_1(t) &= (\theta^2+1)^{-5/2}\theta \\ s_2(t) &= (\theta^2+1)^{-5/2}\theta^2 , \end{aligned} \tag{2.2}$$

where θ is a dimensionless variable

$$\theta = t/\tau \tag{2.3}$$

and τ is a parameter with the dimensions of time

$$\tau = z/v = \text{(smallest slant range)/(closing speed)}. \tag{2.4}$$

In section 1, it was given that $300 \leq z \leq 1500$ ft. and $20 \leq v \leq 70$ kn.; for the surface vessel MAD problem, then, the parameter τ must lie between

$$\tau_{min} = \frac{z_{min}}{v_{max}} = \frac{300 \text{ ft}}{70\text{kn.} \times 1.6878\text{ft/sec/kn}} = 2.54 \text{ sec.} \tag{2.5}$$

and

$$\tau_{max} = \frac{z_{max}}{v_{min}} = \frac{1500 \text{ ft}}{20\text{kn.} \times 1.6878\text{ft/sec/kn}} = 44.4 \text{ sec.} \tag{2.6}$$

For interpretation, these figures mean that the bell-shaped signal $s_0(t)$, equation (2.2), will occur with pulse widths between 2.86 sec. (when $\tau = \tau_{min}$) and 50.2 sec. (when $\tau = \tau_{max}$) at half amplitude, or between 3.64 and 636 sec. at 10% amplitude. For a second interpretive example, "W" signals, $s_2(t)$, will be observed with a "time between peaks" of from 4.15 ($\tau = \tau_{min}$) to 72.5 sec. ($\tau = \tau_{max}$).

Thus an anomaly signal is completely described by

stating 5 numbers: a_0, a_1 and a_2 which govern the signal's shape by equation (2.1), τ, which sets the time scale of the signal, and t_0, the "epoch instant" or time of occurrence, corresponding to t=0 in equations (2.2). But since the purpose of MAD is the detection and localization of suspected anomalies, it is unlikely that any of these 5 quantities can be known in advance. It is not unreasonable, therefore, to describe them as random variables with an unknown, and perhaps time-varying, joint probability distribution. Furthermore, in a tactical encounter it is to be expected that a sequence of anomaly signals with differing values of $\{a_j\}_0^2$, τ, and t_0 will be observed occurring in bursts or clumps, corresponding to repeated detections of the same anomaly.

If the five parameters describing a single signal are regarded as random varaibles, then a sequence of signals may be treated as a random process. This process is likely to be non-stationary, for consider a successful encounter culminating in the localization of a stationary target: as the encounter proceeds in time, the quamtity τ will (on the average) decrease on successive detections as the range, z, decreases. Including nonstationarity in the description of the signal process, however, must ultimately result in the specification of a time-varying filter. For the sake of simplicity in computation and eventual filter construction, the signal process will be taken to be stationary; it is assumed that nonstationarity (like that

in the mythical encounter described above) can be adequately treated in practice by making some filter parameters switchable by the MAD equipment operator. Sufficient conditions for stationarity are (1) the quantities $\{a_j\}_0^2$ and τ have a fixed joint distribution and their values on successive anomaly signals are statistically independent, and (2) the epochs t_0 are uniformly and independently distributed over any finite interval.

For the filter design theory in the next section, the signal process is sufficiently known when its ~~xpx~~ power spectral density is given. Since the epochs are uniform and independent, Campbell's theorem ([3], p. 176) is applicable, with the result that the power spectral density of the process is proportional, at each frequency, to the energy spectral density of a single anomaly signal. Taking the constant of proportionality equal to 1 for convenience, and letting F be the joint c.d.f. (cumulative distribution function) of the random variables $\{a_j\}_0^2$ and τ, the power spectral density $S(\omega)$ of the signal process is given by

$$S(\omega) = \int dF(a_0, a_1, a_2, \tau) \left|\mathcal{F}\{s(t)\}\right|^2 \qquad (2.7)$$

where $\mathcal{F}$ indicates the operation of Fourier transformation, and $s(t)$ is given by equation (2.1). If the additional assumption is made that the a_j are pair-wise uncorrelated, the cross-terms in the integrand of equation (2.7) disappear and $S(\omega)$ can be written

$$S(\omega) = \sum_{j=0}^{2} \bar{a}_j^2 \int |S_j(\omega;\tau)|^2 \, dG(\tau), \tag{2.8}$$

where G is the c.d.f. of τ, and $S_j(\omega;\tau) = \mathcal{F}\{s_j(t;\tau)\}$.

3. Filter design - theory

With the signal and noise models thus far developed, the data-processing problem clearly belongs in the domain of the Wiener-Kolmogorov theory of optimal smoothing. An observation is made of the stationary random process v(t) with spectral density (from equations (1.1) and (2.8))

$$V(\omega) = S(\omega) + N_o/\omega^2 \tag{3.1}$$

(assuming signal and noise are at least uncorrelated), and as faithful a copy of s(t) as possible is to be extracted. The minimum-mean-square-error linear realizable zero-lag smoothing filter $H(j\omega)$ is in principle found by solving an appropriate Wiener-Hopf integral equation; however, by exploiting the relatively simple noise structure $H(j\omega)$ can be found by simpler techniques. If $H(j\omega)$ is the best linear filter for s in s+n, then (by linearity) it is also the best linear filter for $\dot{s}$ in $\dot{s}+\dot{n}$. But here the power spectral density of $\dot{n}$ is $|j\omega|^2(N_o/\omega^2) = N_o$; i.e., $\dot{n}$ is white noise, and the best linear filter for any signal in white noise can be written down by inspection ([4], p. 72 ff.). For the problem at hand, $H(j\omega)$ is the minimum-phase filter whose real part is $\omega^2 S(\omega)$:

$$H(j\omega) = \omega^2 S(\omega) + j\mathcal{H}\{\omega^2 S(\omega)\} \tag{3.2}$$

where $\mathcal{H}$ denotes the operation of Hilbert transformation.

It is therefore necessary to return to equation (2.8) and compute $\omega^2 S(\omega)$; the first step is clearly to find the Fourier transforms $\omega^2 |S_j(\omega;\tau)|^2$ of the signals $\dot{s}_0$, $\dot{s}_1$ and $\dot{s}_2$. Use of the Bateman tables ([5], p. 9) yields

$$\begin{aligned}
\omega^2 |S_0(\omega;\tau)|^2 &= \omega^6 K_2{}^2(\tau\omega) \\
\omega^2 |S_1(\omega;\tau)|^2 &= \omega^6 K_1{}^2(\tau\omega) \\
\omega^2 |S_2(\omega;\tau)|^2 &= \left[\omega^2 K_1(\tau\omega) - \tau\omega^3 K_0(\tau\omega)\right]^2
\end{aligned} \tag{3.3}$$

(up to a common factor). The $K_j(\cdot)$ are modified Bessel functions of the second kind. Now the right-hand sides of equations (3.3) should be inserted into equation (2.8) to compute $\omega^2 S(\omega)$. Unfortunately, neither the mean-square coefficient values $\overline{a_j^2}\ {}^2_0$ nor the time-scale c.d.f. $G(\tau)$ are known.

In order to proceed with the design, an ad hoc technique is employed which, while by no means optimal, is nonetheless plausible and does lead to a comparitively simple design. In brief, $\omega^2 S(\omega)$ is taken to be a bandpass spectrum with its low-frequency behaviour determined by that of $\omega^2 |S_0(\omega;\tau_{max})|^2$ and its high-frequency behaviour determined by that of $\omega^2 |S_0(\omega;\tau_{min})|^2$. (A rationale for this technique is given in Appendix 1 to this report.) With this specification, the desired filter is described, using equation (3.2) by

$$\mathrm{Re}\{H(j\omega)\} = \begin{cases} [(\omega\tau_{max})^3 K_2(\omega\tau_{max})]^2 & \omega \leq \dfrac{1.9}{\tau_{max}} \\ \text{constant} & \dfrac{1.9}{\tau_{max}} < \omega \leq \dfrac{1.9}{\tau_{min}} \\ [(\omega\tau_{min})^3 K_2(\omega\tau_{min})]^2 & \omega > \dfrac{1.9}{\tau_{max}}. \end{cases} \tag{3.4}$$

The number 1.9 is determined by the fact that the function

$x^3 K_2(x)$ assumes a maximum, with slope zero, at $x=1.9$ ([6], p.417).

Of course, a finite RLC circuit cannot be built to satisfy equation (3.4), since the real part of any RLC transmittance must be a rational function of ω^2. Therefore the problem becomes one of approximating equation (3.4) with the real part of an RLC transmittance.

4. Filter design - approximation

Let $\hat{H}(s)$ be the RLC transmittance whose real part approximates equation (3.4); i.e., $\mathrm{Re}\{\hat{H}(j\omega)\} \approx \mathrm{Re}\{H(j\omega)\}$. Then $\hat{H}(s)$ is a rational function of s. Since it is clear (from equation (3.4)) that $\mathrm{Re}\{H(j\omega)\}$ is never negative, it may tentatively be assumed that the degrees of the numerator and denominator of $\hat{H}(s)$ differ by exactly one. Thinking of $\hat{H}(s)$ as a high-pass and a low-pass filter in cascade, it is plausible that the high-pass and low-pass sections may (to a rough approximation) be designed independently of one another since $1.9/\tau_{max}$ and $1.9/\tau_{min}$ (see equation (3.4)) differ by about 1 1/4 decades.

On the low end, $\lim_{x \to 0} x^6 K_2{}^2(x) = 4x^2$ ([6], p.375), so that the minimal high-pass section of H(s) is of the form $\frac{As}{s + p_0}$ because $\lim_{\omega \to 0} \mathrm{Re}\left\{\frac{Aj\omega}{j\omega + p_0}\right\} = \frac{A}{p_0{}^2}\omega^2$. Hence the form of the approximating filter is, as a first guess,

$$H(s) = \frac{As}{s + p_0} \frac{s + z_1}{s + p_1} \frac{s + z_2}{s + p_2} \frac{s + z_3}{s + p_3} \frac{s + z_4}{s + p_4} \frac{1}{s + p_5} \tag{4.1}$$

with the zeros, poles and the factor A to be chosen so as to achieve the best approximation of $\mathrm{Re}\{\hat{H}(j\omega)\}$ to $\mathrm{Re}\{H(j\omega)\}$.

One way of instrumenting the approximation procedure is shown in figure 2. $\hat{H}(s)$ is built with all frequencies

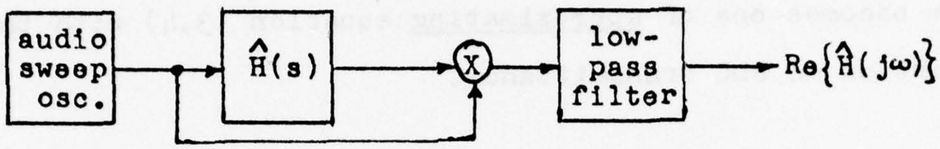


Figure 2

scaled up into the high audio range and with the ten free parameters of equation (4.1) available for adjustment. The system output is displayed on an oscilloscope against a transparency of $\mathrm{Re}\{H(j\omega)\}$ and the parameters A, $\{z_i\}_1^4$ and $\{p_i\}_0^5$ adjusted by hand to give a "good" approximation. Not only are the equipment requirements to implement this scheme quite stringent (e.g., the low-pass filter must be tunable to remove the second harmonic of the oscillator as it is swept over several decades), but the adjustments of the ten parameters would be interdependent rendering a good approximation difficult to find.

The approximation method finally chosen employed digital computation.* $\mathrm{Re}\{H(j\omega)\}$ was sampled at 94 points over the frequency range 0 to approximately 2 c/s, and a digital computer was programmed to employ a modified Newton's technique that varied the parameters in $\hat{H}(s)$, starting from a set of initial estimates, so as to minimize the squared error

$$\varepsilon^2 = \sum_{k=1}^{94} \left[\mathrm{Re}\{\hat{H}(j\omega_k)\} - \mathrm{Re}\{H(j\omega_k)\}\right]^2. \qquad (4.2)$$

*A description of the method is given in Appendix 2.

Several different sets of initial parameter estimates were used, in order to make more likely the achievement of a global, rather than just a local, minimum for ε^2. The 4-zero, 5-pole filter that resulted was deemed to have an unacceptably large squared error; furthermore, its 3 db bandwidth was approximately 2.2 decades. Since the 3 db bandwidth of a single bell-shaped signal $s_0(t)$ is only about 0.6 decades, it was felt that a 2.2 decade wide filter would pass so much noise that weak-signal performance would be very poor.

Since existing MAD equipment is provided with two display channels, it was decided to cover the frequency range of interest with two filters, the first for signals having values of the time-scale parameter τ between $\tau_{min,HF} = 2.54$ and $\tau_{max,HF} = 10.6$ seconds, and the second for the range between $\tau_{min,LF} = 10.6$ and $\tau_{max,LF} = 44.4$ seconds. Two filters having the form of equation (4.1) were designed using the computer program mentioned above. After some experimentation, it was observed that a considerable reduction in minimum squared error was achieved by deleting the numerator factor $(s + z_4)$ from each filter. This is reasonable, in retrospect, since the real part of each filter function becomes negative at the high end of its passband allowing a better fit to $Re\{H(j\omega)\}$ which, though always positive, decreases exponentially to zero at high frequencies. It was also noted that the minimum squared errors of the best 5-pole, 3-zero filters were

not significantly increased by deleting a factor $(s+z_3)/(s+p_5)$ from each filter and optimizing.

5. Filter construction and estimated performance

The designs finally arrived at are given by

$$\hat{H}_{LF}(s) = \frac{1.75\ s\ (s+.574)}{(s+.0213)(s+.459)(s+.592)(s+.787)}$$

$$\hat{H}_{HF}(s) = \frac{30.24\ s\ (s+2.21)}{(s+.0898)(s+1.861)(s+2.388)(s+3.206)}\ . \tag{5.1}$$

Because of the extremely low frequencies involved, the use of inductance in the construction of the filters is impractical. However, it is well known that any RLC transmittance can be realized with RC circuits only in the forward and feedback paths of an ideal operational amplifier; this method of synthesis is certainly appropriate here. The squared error between $Re\{\hat{H}\}$ and $Re\{H\}$ is at a minimum when the filter poles and zeros are those given in equations (5.1). From a practical standpoint, though, the squared error is moderately insensitive to small errors in the poles and zeros; 5% tolerance is probably adequate. The squared error is least insensitive to the lowest-frequency pole in the two filters ($s = -.0213$, $s = -.0898$), because the noise power density is highest at low frequencies. But this fact is relatively unimportant, since it is recommended in section 6 that these poles be adjustable anyway. Values of the gain factors (1.75 and 30.24) are of course unimportant; indeed it can be shown that with these gain factors and an

input noise spectral density N_0/ω^2, the r.m.s. output noise from the HF filter is slightly less than one-half the r.m.s. output noise from the LF filter. Thus these gain factors should be individually adjustable. In practice, the gains would probably be set so that the r.m.s. noise outputs from the two channels were approximately equal.

An alternative synthesis is implied by expanding equations (5.1) in partial fractions:

$$\hat{H}_{LF}(s) = \frac{-.1077}{s+.0213} + \frac{4.8378}{s+.459} + \frac{1.2599}{s+.592} + \frac{-5.990}{s+.787}$$
$$\hat{H}_{HF}(s) = \frac{-.4539}{s+.0898} + \frac{15.6442}{s+1.861} + \frac{12.9743}{s+2.388} + \frac{-28.1646}{s+3.206} . \tag{5.2}$$

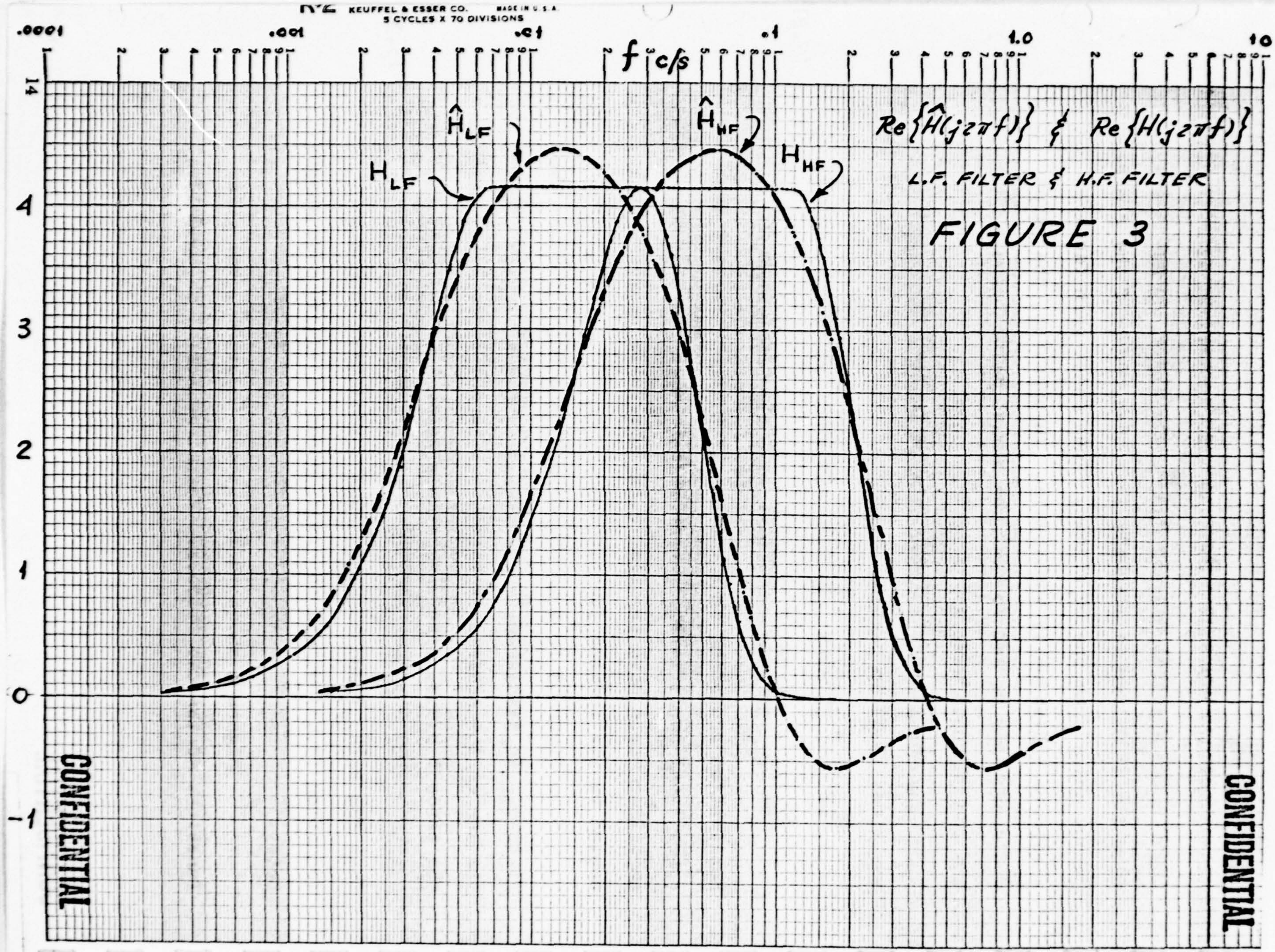
Realization according to equations (5.2) would require two operational amplifiers for each filter; each operational amplifier would have two simple RC integrator networks connected to its summing junction.

Figure 3 is a graph of $Re\{\hat{H}\}$ and $Re\{H\}$ for the two filters. In each case the squared error is $\varepsilon^2 \approx 8.1$. Since the sum of the squares of the 94 sample points of $Re\{H\}$ is 833, the percent r.m.s. error per sample point is

$$\varepsilon_{\%} = \sqrt{\frac{8.1/94}{833/94}} \times 100 \doteq 10\%. \tag{5.3}$$

That is, $Re\{\hat{H}_{LF}\}$ and $Re\{\hat{H}_{HF}\}$ match the desired response (equation (3.4)) within an r.m.s. error of 10% at each sample point.

Figure 4 shows $|\hat{H}_{LF}(j\omega)|$ and $|\hat{H}_{HF}(j\omega)|$, the gain functions of the two filters. Their (common) 3 db bandwidth



Re{Ĥ(j2πf)} & Re{H(j2πf)}
L.F. FILTER & H.F. FILTER
f c/s
Ĥ_LF
H_LF
Ĥ_HF
H_HF
KEUFFEL & ESSER CO. MADE IN U.S.A.
5 CYCLES X 70 DIVISIONS

FIGURE 3

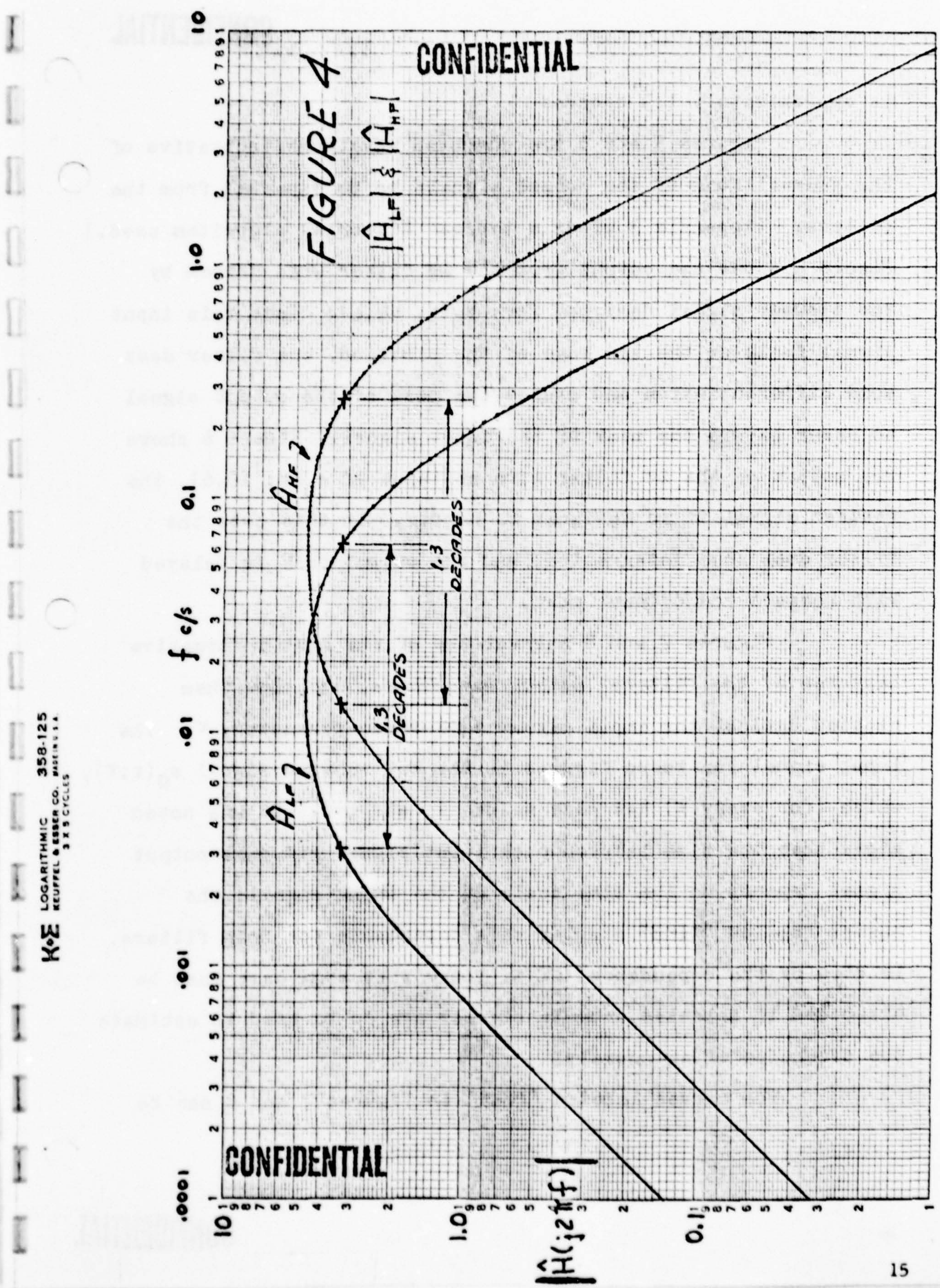

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FIGURE 4
$|\hat{H}_{LF}|$ & $|\hat{H}_{HF}|$
A_{HF}
A_{LF}
1.3 DECADES
1.3 DECADES
f c/s
.0001
.001
.01
0.1
1.0
10
$|\hat{H}(j2\pi f)|$
10
1.0
0.1
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K+E LOGARITHMIC 358-125
KEUFFEL & ESSER CO. MADE IN U.S.A.
3 X 5 CYCLES

is approximately 1.3 decades.

Figures 5 and 6 are computed waveforms indicative of the general form of the output signals to be expected from the filters. (Appendix 3 gives a typical filtering algorithm used.) Figure 5 shows the output from the LF filter when driven by the slowest signal designed for: $s_0(t, 44.4)$. Since this input signal falls at the low edge of the passband, the filter does some differentiation and causes the peak of the output signal to occur <u>before</u> the peak of the input signal. Figure 6 shows the output of the LF filter with an input of $s_0(t; 10.6)$, the fastest signal it is designed to handle. In this case the filter does some integration, and the output peak is delayed with respect tothe input peak.

Figures 7 and 8 extract two of the most informative features of the filters' output waveshapes, and show them plotted against the input signal time-scale parameter, τ. The input signal for these figures is the bell-shaped signal $s_0(t;\tau)$, which (for every τ) has peak value 1 at t=0. It was noted above that the time between peak input signal and peak output signal depends on the time scale of the inptt signal; the curves labeled "TIME" display this dependence for both filters. In view of the dependence, it is clear that some care must be exercised if detected anomaly signals are to be used to estimate the direction of an anomaly.

The curves labeled "PEAK" in figures 7 and 8 can be

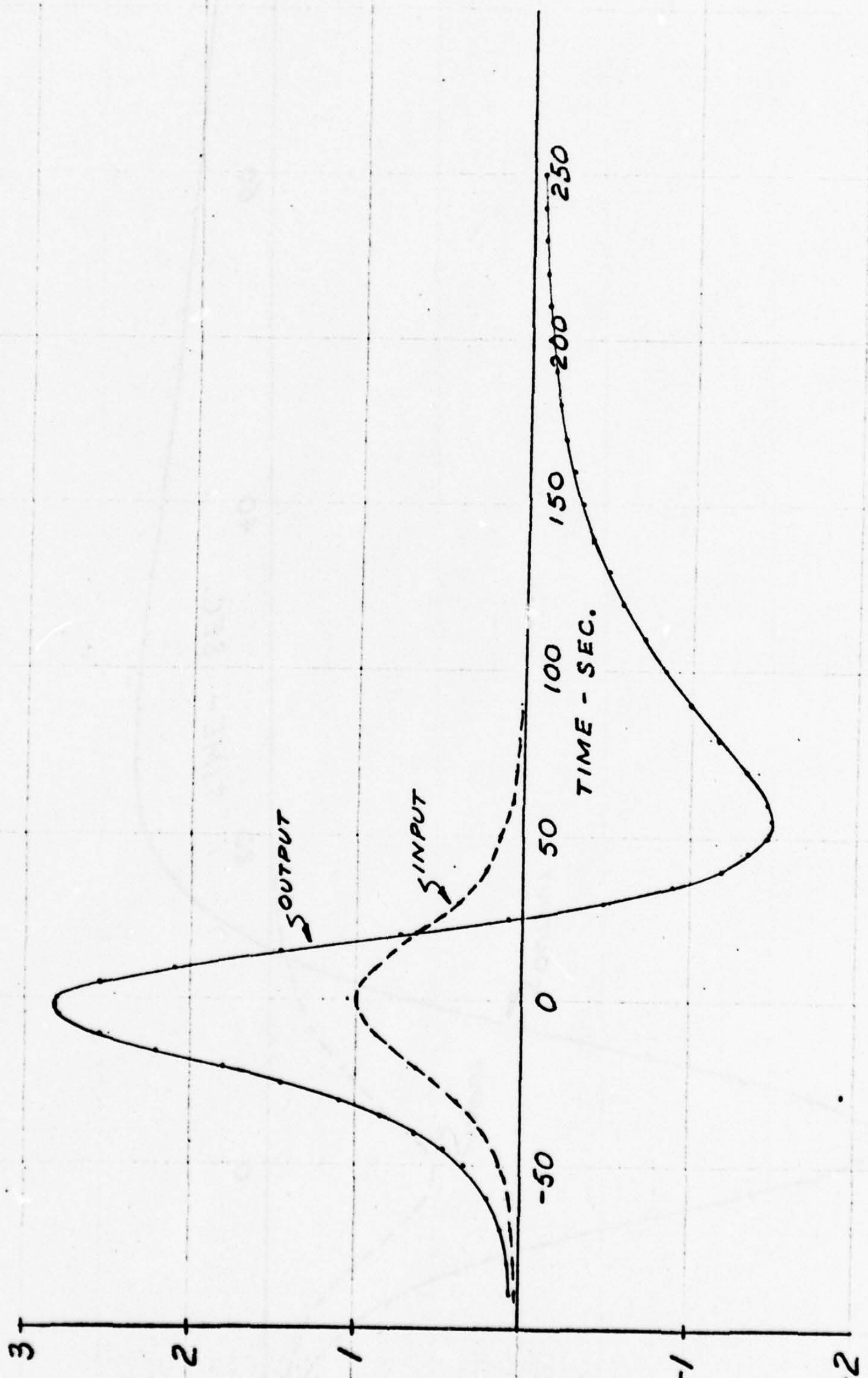

FIG. 5 LF FILTER – OUTPUT FOR $S_0(t)$ INPUT, τ = 44.4 SECONDS.

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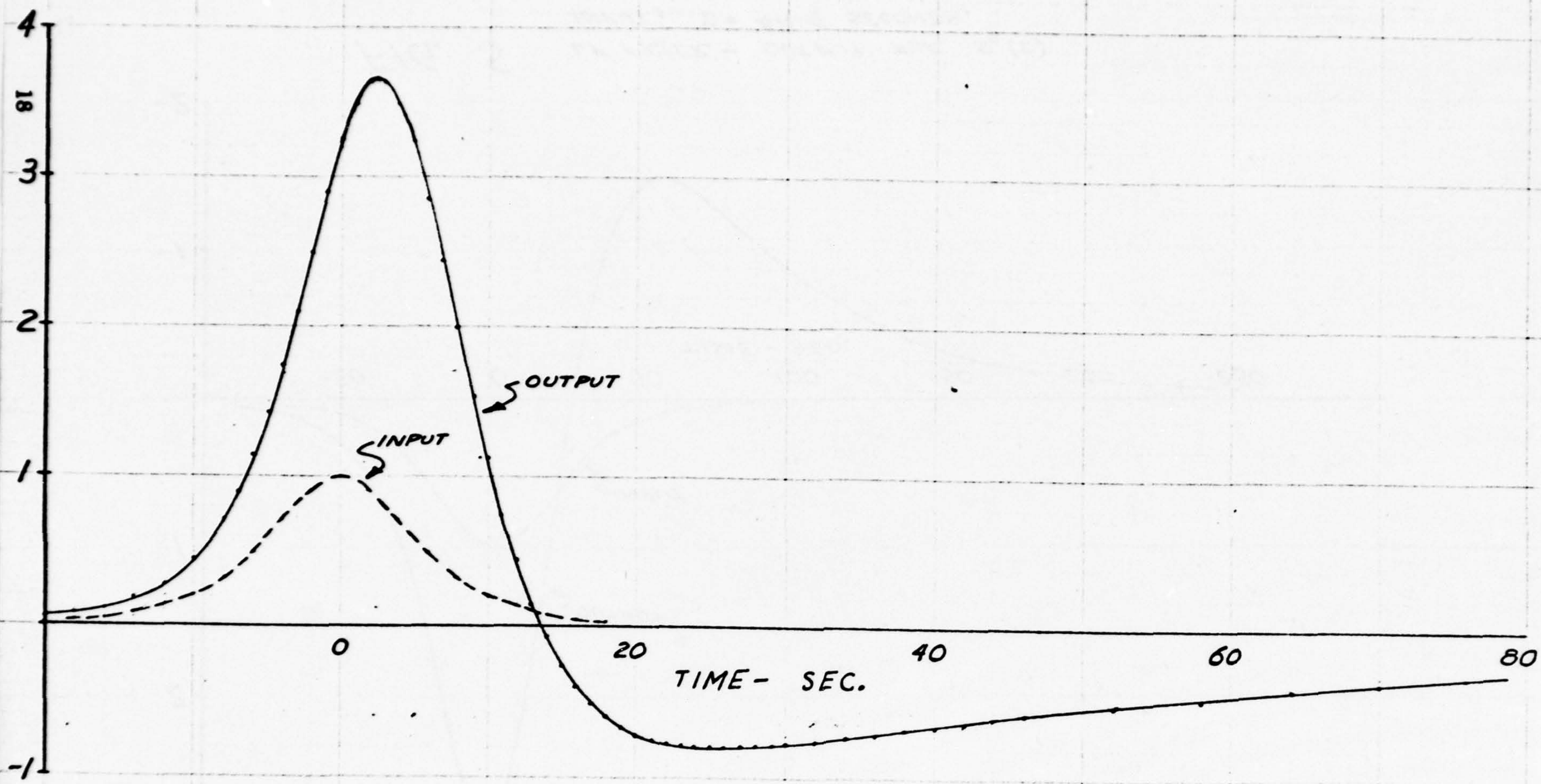

FIG. 6 LF FILTER - OUTPUT FOR $S_0(t)$ INPUT, $\tau = 10.6$ SECONDS.

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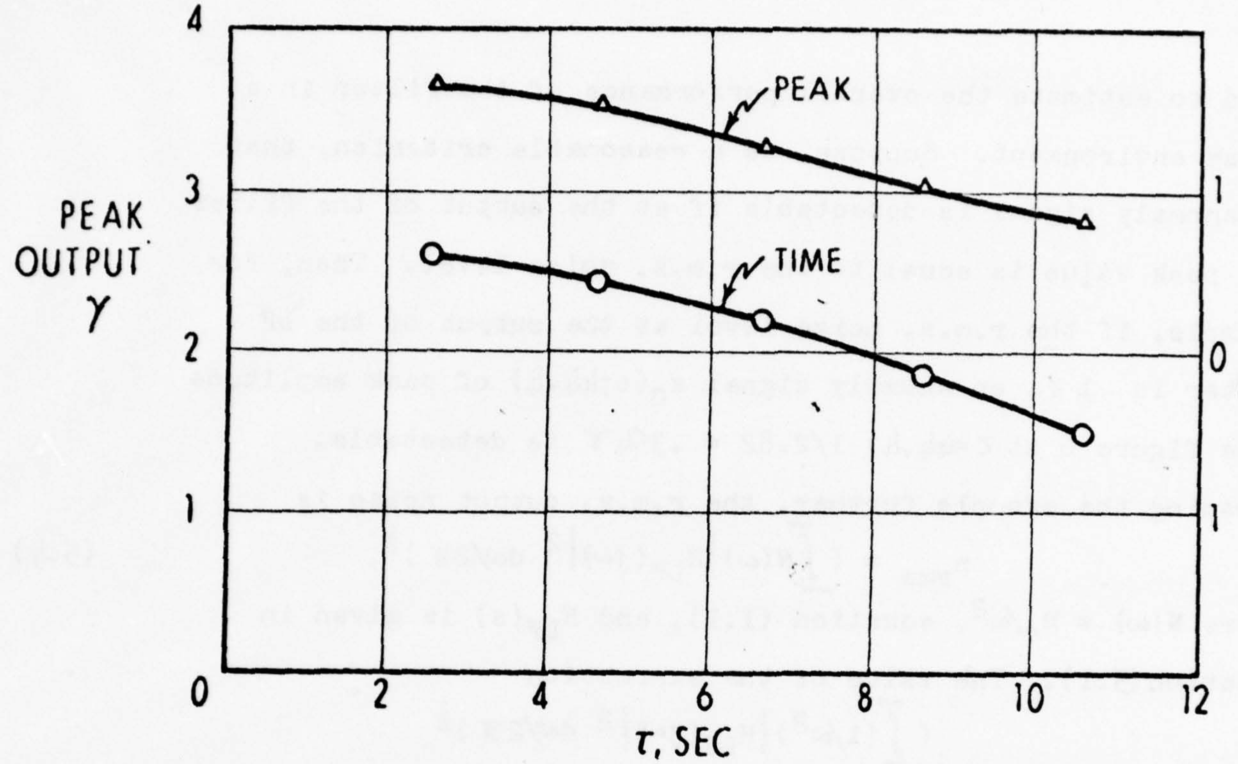

Fig. 7 HF Filter - Peak Output for 1γ Peak Input and Time from Input Peak to Output Peak

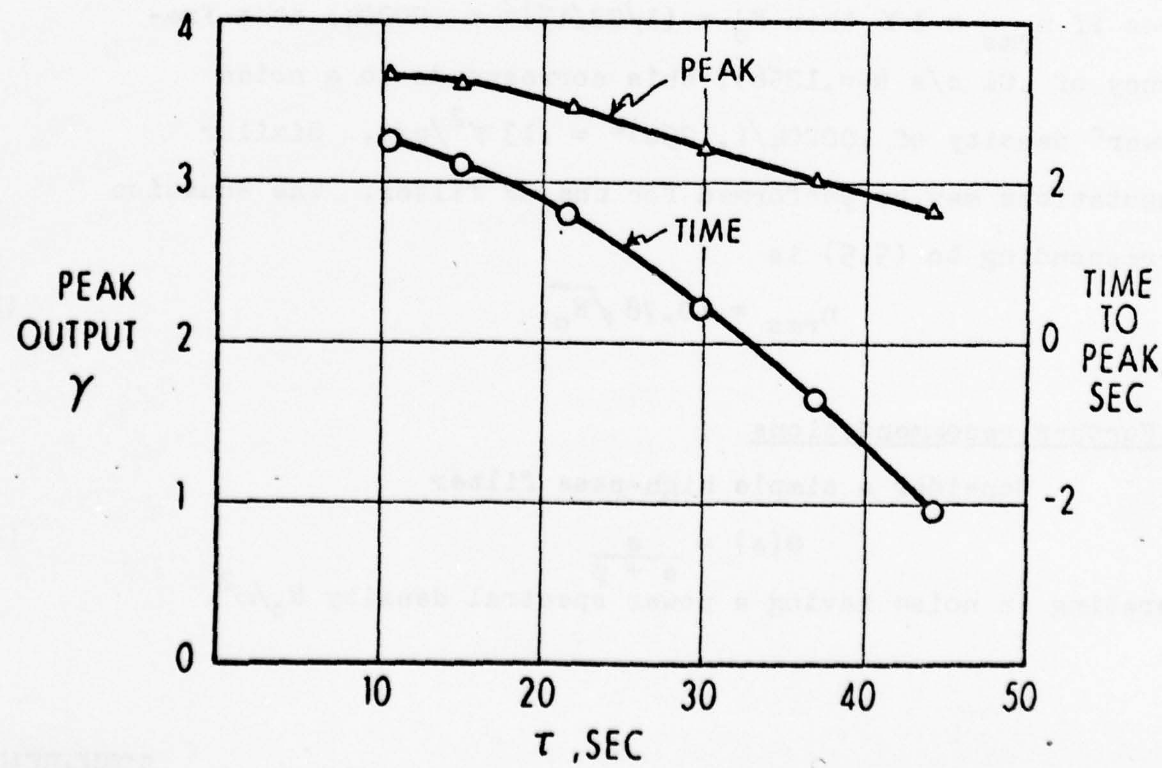

Fig. 8 LF Filter - Peak Output for 1γ Peak Input and Time from Input Peak to Output Peak

used to estimate the overall performance of the filter in a noisy environment. Suppose, as a reasonable criterion, that an anomaly signal is detectable if at the output of the filter its peak value is equal to the r.m.s. noise level. Then, for example, if the r.m.s. noise level at the output of the LF filter is $1\,\gamma$, an anomaly signal $s_0(t;44.4)$ of peak amplitude (see figure 8 at $\tau = 44.4$) $1/2.82 = .354\,\gamma$ is detectable. Pursuing the example further, the r.m.s. output noise is

$$n_{rms} = \left(\int_{-\infty}^{\infty} N(\omega) \left| H_{LF}(j\omega) \right|^2 d\omega/2\pi \right)^{\frac{1}{2}}, \tag{5.4}$$

where $N(\omega) = N_0/\omega^2$, equation (1.1), and $H_{LF}(s)$ is given in equation (5.1). The value of the expression

$$\left(\int_{-\infty}^{\infty} (1/\omega^2) \left| H_{LF}(j\omega) \right|^2 d\omega/2\pi \right)^{\frac{1}{2}}$$

is readily computed to be 22.15, so that

$$n_{rms} = 22.15 \sqrt{N_0}. \tag{5.5}$$

Hence if $n_{rms} = 1\,\gamma$ then $N_0 = (1/22.15)^2 = .00204$; at a frequency of .01 c/s ($\omega = .1256$), this corresponds to a noise "power" density of $.00204/(.1256)^2 = .13\,\gamma^2/\text{c/s}$. Similar computations may be performed for the HF filter. The equation corresponding to (5.5) is

$$n_{rms} = 10.78 \sqrt{N_0}. \tag{5.6}$$

6. Further recommendations

Consider a simple high-pass filter

$$G(s) = \frac{s}{s + p} \tag{6.1}$$

operating in noise having a power spectral density N_0/ω^2.

A simple calculation similar to equation (5.4) shows that the r.m.s. output noise level from G is

$$n_{rms} = \sqrt{N_o/2p}. \qquad (6.2)$$

Thus doubling the low-frequency cutoff from p to 2p reduces n_{rms} by a factor of $\sqrt{2}$, or 3 db. Because the low- and high-frequency cutoffs of the filters discussed in this report (equations (5.1)) differ by better than 1.3 decades, the variation in r.m.s. output noise level from $\hat{H}_{LF}$ or $\hat{H}_{HF}$ with moderate variations in location of the low-frequency cutoff (.0213 or .0898) will be approximately as given in equation (6.2).

The conjecture was made in section 4 that the low- and high-frequency characteristics of $\hat{H}_{LF}$ and $\hat{H}_{HF}$ could be designed independently. While this is not strictly true, still it is a useful first approximation. Therefore if τ_{max} were halved, from 44.4 to 22.2 sec., corresponding to a change in the the original specifications of section 1 to z_{max}=750 ft. or v_{min}=40kn. (see equation (2.6)), only the low-frequency part of H_{LF} (equation (3.4)) would change; the principal effect on $\hat{H}_{LF}$ would most probably be simply to change the location of the lowest-frequency pole from s=-.0213 to s=-.0426. As noted above, such a change would result in a 3 db decrease in r.m.s. output noise level from $\hat{H}_{LF}$. For an anomaly signal actually occurring at the <u>high</u> end of $\hat{H}_{LF}$'s bandpass (say at τ = 10.6), the change in the low-frequency cutoff would have an insignificant effect

on the peak output signal. Hence such a signal would be observed with an output signal-to-noise ratio increased by 3 db.

The remarks and calculations above confirm in a quantitative way what is already intuitively known: when trying to detect a moderately fast but possibly weak anomaly signal, an enormous premium is paid in output signal-to-noise ratio for the ability to see slow, but very strong, signals.

It is therefore recommended that the low-frequency pole of $\hat{H}_{LF}$ be adjustable over a range in absolute value from .0213 upwards in several steps to perhaps 2 octaves higher. The adjustment upward from .0213 would be made by the MAD equipment operator whenever it is known that his target is moderately near or closing moderately fast. There seems little point in making adjustments available over a wider range than two octaves, because $\hat{H}_{LF}$ would then be approximately designed for an empty set of τ values (recall $\tau_{max,LF} = 44.4$, $\tau_{min,LF} = 10.6$, and two octaves from 44.4 is 11.1). That is, any signal observable at the output of $\hat{H}_{LF}$ with its low-frequency cutoff more than two octaves above .0213 is bound also to be seen at the output of $\hat{H}_{HF}$.

It is possible that under high ambient noise conditions (large values of N_o, equation (1.1)) the output from $\hat{H}_{LF}$ would become totally unusable, and even the output of $\hat{H}_{HF}$ would be very noisy. Thus it seems reasonable to provide one or two adjustable low-frequency cutoffs for $\hat{H}_{HF}$ as well; it is expected,

however, that these would be rarely used.

Switchable cutoffs are, of course, provided on present-day airborne MAD equipment. The low-frequency roll-off of these filters is much faster than the 6 db/octave of the filters proposed in this report, and it is conjectured that the reason for this is to minimize noise from periodic maneuvers that falls just below the airborne MAD passband. For shipborne MAD, periodic maneuver noise will fall within the passband of $\hat{H}_{LF}$ and $\hat{H}_{HF}$, and should be removed by notch filters as recommended in section 1.

It is <u>not</u> recommended that the high-frequency cutoffs of $\hat{H}_{LF}$ and $\hat{H}_{HF}$ be adjustable. The decrease of noise power density with increasing frequency, equation (1.1), is so rapid that the resulting increase in output signal-to-noise ratio from such provision is insignificant. To support this statement, a filter was optimized by the algorithm in Appendix 2 for the single signal $s_0(t; 44.4)$. It would be expected to produce a higher output signal-to-noise ratio (with $s_0(t; 44.4)$ as input) than $\hat{H}_{LF}$ with its high-frequency cutoff moved downwards in frequency. Yet the filter optimized for $s_0(t; 44.4)$ yields a signal-to-noise ratio less than $\frac{1}{4}$ db better than $\hat{H}_{LF}$ <u>unmodified</u>.

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5. A. Erdelyi (ed.) "Tables of Integral Transforms", Vol. I, McGraw-Hill (1954)

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Appendix 1 - Derivation of Equations (3.4)

Equations (3.4) were derived by first normalizing the (bandpass) spectra of $\dot{s}_0$, $\dot{s}_1$ and $\dot{s}_2$ so that the maxima of the spectra were all the same, and independent of τ. Then at each ω, $\mathrm{Re}\{H(j\omega)\}$ was taken to be the supremum (over a_0, a_1, a_2 and τ) of these normalized spectra. Since for any fixed τ, $\dot{s}_0$ occupies a wider frequency band than either $\dot{s}_1$ or $\dot{s}_2$, equations (3.4) result. This procedure is a sort of "worst case" design that ensures $H(j\omega)$ is wide enough to pass every anomaly signal under consideration.

Since no information is available about the relative mean-square values of the coefficients a_0, a_1 and a_2, it is not unreasonable to permit the design to be dominated by the widest band signal, s_0.

The treatment of the unknown time-scale c.d.f. $G(\tau)$ is perhaps less satisfactory. It might be thought that without a priori knowledge of G that the so-called "principle of minimum prejudice" (PMP) should be invoked. The PMP asserts that the minimum prejudice is inserted into the problem by the designer if $G(\tau)$ is chosen to be the distribution with the maximum possible entropy

$$H[G] = \int g(\tau) \log g(\tau)\, d\tau \qquad \text{(A1.1)}$$

consistent with the constraints of the problem, where $g(\tau) = dG(\tau)/d\tau$. For the problem at hand, the result would be that τ should be assumed uniformly distributed over the interval

(2.54, 44.4). It was felt that this choice for $G(\tau)$ is inappropriate, and the PMP is not applicable so simply. Assuming τ to be uniformly distributed would certainly result in a narrower filter than the filter specified by equations (3.4), and its performance on signals with extreme values of τ would necessarily be degraded. Use of the PMP implies that some measure of filter performance is averaged over all values of τ; poor performance at extremes of the range is more than compensated by improvements at moderate τ values. But the design specifications of section 1 do not admit such an averaging process; the filter is to be "good" for each value of τ individually in the interval (2.54, 44.4).

The choice of equations (3.4) to specify $\mathrm{Re}\{H\}$ is by no means unique, and its optimal properties, if any, are ill-defined. Nonetheless it has the virtue of simplicity in its formulation and in the final filter design to which it leads.

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Appendix 2 - Parameter Optimization

```
begin integer samplesize; read(samplesize);

begin integer iter, maxiter, halve, maxhalf, i; real a, error, newerror;

real array b, newb, delta, partials[1:5], w,y,mag,angle,realpart[1:samplesize];

procedure finderror(polesandzeros, frequencies, givenfunction, samplesize,

magnitude, angle, realpart, a, error);

     comment: inputs are the 1:5 array of the zero and four poles, the
          frequencies at which the given function is sampled, and the sample
          values of the given function.  outputs are magnitude, angle and
          real part of the approximating function at the sample points, the
          scale factor a, and the squared error;

procedure derive(polesandzeros, frequencies, givenfunction, samplesize,

magnitude, angle, realpart, a, partials);

     comment: all arguments are inputs except partials, which is a 1:5
          array of the partial derivatives of the squared error with respect
          to the elements of polesandzeros;

comment: now begin program by reading two parameters that control the iterative
     loops, first guesses of the zero and four poles, sample frequencies and
     sample values;

read(maxiter, maxhalf, for i:= 1 step 1 until 5 do b[i], for i:= 1 step 1

     until samplesize do begin w[i], y[i] end);

finderror(b, w, y, samplesize, mag, angle, real, a, error);

for iter:  0 step 1 until maxiter do

beginderive(b, w, y, samplesize, mag, angle, real, a, partials);

     for i:= 1 step 1 until 5 do

     begin delta[i] := -error/(8*partials[i]);

          testplus: if b[i] + delta[i] > 0 then go to nexti;

          delta[i] := delta[i]/2; go to testplus;

          nexti: newb[i] := b[i] + delta[i]

     end for i;

     for halve:= 0 step 1 until maxhalf do
```

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```
    begin finderror(newb, x, y, samplesize, mag, angle, real, a, newerror);
          if newerror < error then
          begin error:= newerror;
                for i:= 1 step 1 until 5 do b[i]:= newb[i];
                go to presson
          end then;
          for i:= 1 step 1 until 5 do newb[i]:= (newb[i] + b[i])/2
    end for halve; go to panicstop;
    presson: print(a, error, for i:= 1 step 1 until 5 do b[i])
end for iter; go to programend;
panicstop: print("CANNOT REDUCE ERROR AT MINIMUM DELTAS");
print(a, error, for i:= 1 step 1 until 5 do b[i]);
print(a, newerror, for i:= 1 step 1 until 5 do newb[i]);
programend: end end
```

Note: In the interests of readability, certain obvious and/or conventional features of the program have been deleted. These include the bodies of the procedures finderror and derive, some calls by value, and formatting for the print and read procedures.

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Appendix 3 - Filter Algorithm

```
begin integer i,n; real input, output, lastoutput, peaktime;
real array decrement[1:4], residue[1:4], y[1:4]; boolean peakfound;
comment: convolves s_o(t), tau = 44.4, with lf filter using step size
      0.1 second.  prints output every 5 sec. from -140 sec to 250
      sec.. locates peak output to 0.1 sec.;
decrement[1] := exp(-.00213);      residue[1] := -.1077;
decrement[2] := exp(-.0459);       residue[2] := 4.8378;
decrement[3] := exp(-.0592);       residue[3] := 1.2599;
decrement[4] := exp(-.0787);       residue[4] := -5.990;
print("TIME (SEC.)", "INPUT", "OUTPUT");
for i := 1 step 1 until 4 do y[i] := 0; peakfound := false;
lastoutput := 0;
for n := -1400 step 1 until 2500 do
    begin output := 0; input := (1 + (n/444)↑2)↑(-5/2);
    for i := 1 step 1 until 4 do
        begin y[i] := y[i] * decrement[i] + input;
        output := output + y[i] * residue[i]/10
        end for i;
    if peakfound then go to tryprint else if output > lastoutput then
    go to bump;
    peakfound := true; peaktime := (n-1)/10; go to tryprint;
    bump: lastoutput := output;
    tryprint: if n/50 ≠ n÷50 then go to loop;
    print(n/10, input, output);
    loop: end for n;
print("PEAK OUTPUT", lastoutput, "AT TIME", peaktime)
end program
```


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Appendix 4 - Glossary of Terms

$N(\omega)$	- noise power spectrum
$s(t)$	- anomaly signal of a submarine
$\dot{s}(t)$	- time derivative of $s(t)$
$s_o(t)$, $s_1(t)$, $s_2(t)$	- three classes of submarine signals comprising $s(t)$
$\dot{s}_o(t)$, $\dot{s}_1(t)$, $\dot{s}_2(t)$	- time derivatives of $s_o(t)$, $s_1(t)$, $s_2(t)$
τ	- ratio of smallest slant range to closing speed
$S(\omega)$	- signal power spectrum
$V(\omega)$	- observed signal power spectrum (signal plus noise)
$H(j\omega)$	- optimum linear filter
$K_j(\cdot)$	- modified Bessel functions of the second kind
$\widehat{H}(j\omega)$	- lumped parameter approximation to $H(j\omega)$

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Report 537/66
LINEAR FILTERING FOR SHIPBOARD MAD RECORDERS (U), CONTRACT 61533-2551-65, by Stephen S. Wolff. Aug 1966. 30 pp. Figs. ~~CONFIDENTIAL~~

This report describes the derivation of linear filters for MAD equipment to be used for submarine detection from high-speed surface vessels. Since the bandwidth (over)

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2. Recorders - Filtering
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UNITED STATES NAVY
MARINE ENGINEERING LABORATORY
ANNAPOLIS, MARYLAND — 21402

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IN REPLY REFER TO:

NP/9690 (631 WWA)
Rept 537/66
Ser: 0276

28 FEB 1967

From: Commanding Officer and Director
To: Commander, Naval Ship Systems Command (SHIPS 1622F)

Subj: MEL Report 537/66, Transmittal of

1. Transmitted herewith is MEL Report 537/66, Linear Filtering for Shipboard MAD Recorders (U), NAVSHIPS Sub-project S-F101 03 21, Task 1522.

2. NAVSHIPS Sub-project S-F101 03 21, Task 1522, entitled "Shipboard ASW Magnetometer Systems," is an exploratory development effort aimed at determining the feasibility of placing magnetometers aboard high speed surface craft for ASW search and classification purposes.

3. At the beginning of the project, it was realized that the closing speeds between the search vessel and the target were radically different from those encountered with aircraft MAD gear, thus necessitating redesign of the magnetometer filter system. Approximate filters were readily derived, but a more precise filter was required for optimum signal processing in a geomagnetic noise environment. To accomplish the detailed analysis, Navy Contract 61533-2551-65 was let to Dr. Stephen S. Wolff at Johns Hopkins University, a recognized expert in the signal processing field.

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NP/9690 (631 WWA) MEL Rept 537/66

4. The enclosed report is the output of this contract. It is mathematical in nature based upon reasonable assumptions on physical parameters which will later be confirmed or modified by analysis of geomagnetic noise data obtained during the summer of 1966 at Wallops Island, Virginia. The main conclusion of the report, i.e., that a dual output, two band-pass filter is required, has been evaluated at MEL and will be incorporated into future systems provided that the maneuver noise encountered aboard the craft carrying the sensor does not radically affect filter design.

5. This letter is unclassified upon removal of Report 537/66.

R. J. Wylde

R. J. WYLDE
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